ANALYTICAL GEOMETRY

Points to be covered in this topic

1. INTRODUCTION
2. SIGN OF CO-ORDINATES
3. DISTANCE FORMULAE
4. STRAIGHT LINE
5. SLOPE OR GRADIENT OF A STRAIGHT LINE
6. CONDITIONS FOR PARALLELISM AND PERPENDICULARITY OF TWO LINES
7. SLOPE OF A LINE JOINING TWO POINTS
8. SLOPE – INTERCEPT FORM OF A STRAIGHT LINE

❑ INTRODUCTION

- Straight line is a curve such that **all points of the line segment** joining **any two points on it** lie on it.
- In analytic geometry a line in the plane is often defined as the set of points **whose co-ordinates satisfy a given liner equation.**

❖ Equation of lines parallel to the co-ordinates axes

- Parallel to x-axis: Equation of a line parallel to x-axis at a distance k from it is $y = k$ if the line below to x-axis at a distance k, then its equation is $y = -k$. (fig. 1)

- Parallel to y-axis: Equation of a straight line parallel to y-axis at a distance from y axis is $x = a$ or $x = -a$.

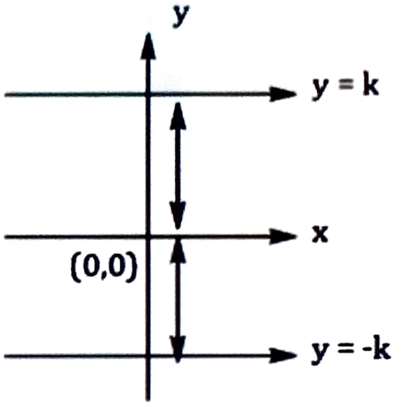


fig. 1

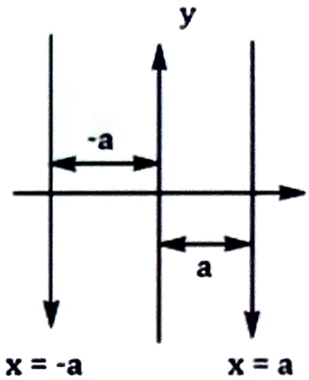
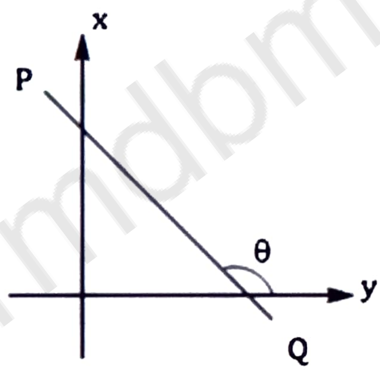


fig. 2

❑ SLOPE OR GRADIENT OF A STRAIGHT LINE

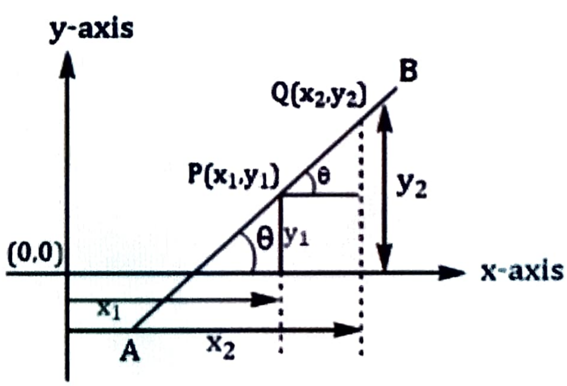
- The slope of a line is the tangent of the angle which the line PQ makes with the +ve direction of the x-axis and it is denoted by m.



1. **θ is positive** if it is measured in **anticlockwise direction** from x and it is negative if it is **measured in clockwise direction**
2. The angle of **inclination of a line with positive direction** of x-axis in anticlockwise direction always lies between 0° and 180°

❑ SLOPE OF A LINE JOINING TWO POINTS

- Let P (x_1,y_1) and Q(x_2y_2) are two points on the line AB



- The slope of a line is the tangent of the angle which the line PQ makes with the +ve direction of the x-axis and it is denoted by m.
- The slope of AB is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} \text{ or } m = \frac{y_1 - y_2}{x_2 - x_1} = \frac{\text{Change in y-co-ordinate}}{\text{Change in x-co-ordiate}}$$

in triangle PRQ

$PR = x_2 - x_1$

$QR = y_2 - y_1$

$$\tan\theta = \frac{QR}{PR} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

❑ CONDITIONS FOR PARALLELISM AND PERPENDICULARITY OF TWO LINES

1. Let $y = m_1x + c_1$ and $y = m_2x + c_2$ are two given lines, they will be parallel if the angle q is between them is either 0° or 180°

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1m_2}$$

if $0 = m_1 - m_2 \Rightarrow m_1 = m_2$

or we can say that if $m_1 = m_2$ (Slope are same)

thus the lines $a_1x + b_1y + c_1 = 0$ and

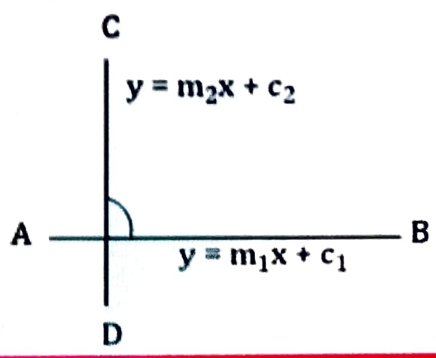
$a_2x + b_2y + c_2 = 0$ will be parallel if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

coefficient of x and y are in proprtional.

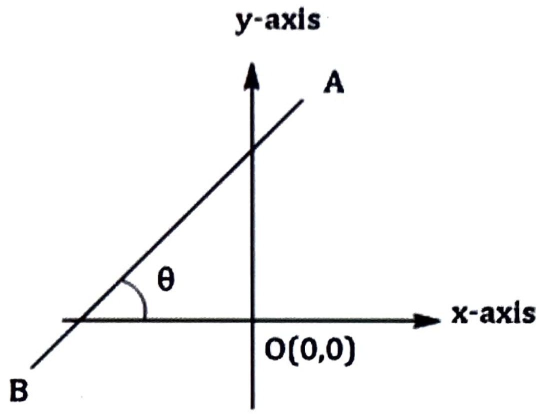
2. The two lines $y = m_1x + c_1$ and $y = m_2x + c_2$ will be perpendicular if, the angle between them is 90 °, tan q = tan 90 = , which possible, when $1 + m_1m_2 = 0$ or $m_1m_2 = -1$, The product of their slopes is equal to -1, thus, the lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular if

$$\left(-\frac{a_1}{b_1}\right)\left(-\frac{a_2}{b_2}\right) = -1 \text{ or } a_1a_2 + b_1b_2 = 0$$



❑ SLOPE INTERCEPT FROM OF LINE

- Let AB is a line, which makes an angle θ with the +ve direction of x axis and cut an intercept c on +ve y axis. Then the equation of the line AB is given by, $y = mx+c$, where $m = \tan\theta$ is called slope intercept from a line



Question: Find the equation of a line which cut off an intercept of 3 unit on negative direction of y axis and makes an angle of 120° with the positive direction of x-axis.

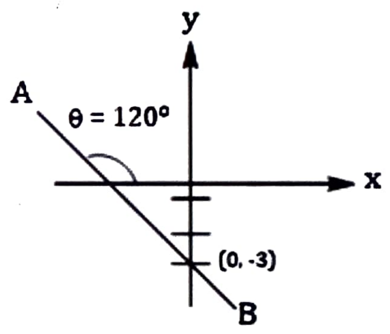
$m = \tan\theta = \tan\ 120° = -\sqrt{3}$

and c = -3 (intercept on y axis)

so the equation of line is $y = mx+c$

$y = -\sqrt{3x} + (-3)$

or $y + \sqrt{3x} + 3 = 0$



Question: Find the equation of the straight line which makes equal intercept on the axes and passes through the point (3, -5)

$\frac{x}{h} + \frac{y}{h} = 1$

this line is passing through (3, -5)

$\frac{3}{h} + \frac{(-5)}{h} \Rightarrow h = -2$

hence equation of required line is

$x+2 = -2$

$y = x+2$ (Slope intercept from m = 1, c = 2)

Question: Find the angle between lines $x\text{-}y\sqrt{3}-5=$ and $x\sqrt{3}+y\text{-}7=9$

$x\text{-}y\sqrt{3}-5=$ and $x\sqrt{3}+y\text{-}7=9$

The equation of given lines are

$x\text{-}\sqrt{3y}-5=0$

$\sqrt{3x}+\ y\text{-}7=9$

The equation can be written in standard form ($y=mx+c$)

$y=\frac{1}{\sqrt{3}}\times\frac{5}{\sqrt{3}}$

$y=\text{-}\sqrt{3}x+7$

Here $m_1=\frac{1}{\sqrt{3}}$

$m_2=\text{-}\sqrt{3}$

Clearly $m_1.m_2=\left(-\frac{1}{\sqrt{3}}\right)\left(\sqrt{3}\right)=\ \text{-}1$

Both the lines are perpendicular to each other

Question: Find the equation of a line passing through (1,1) and perpendicular to the line 3x-4y = 6.

Let AB is a line perpendicular to the given line $3x\text{-}4y=6$

The equation of AB is given by

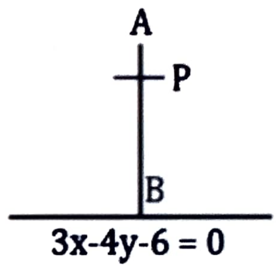
$3y+4x+\lambda=0$

the line is passing through (1,1)

$3+4+\lambda=0$

$\lambda=\text{-}7$

$3y+4x=7$



INTEGRATION

Points to be covered in this topic

1. INTRODUCTION
2. STANDARD FORMULAE
3. METHOD OF SUBSITITUTION
4. METHOD OF PARTIAL FRACTION
5. INTEGRATION BY PARTS
6. DEFINITE INTEGRALS
7. APPLICATION

INTRODUCTION

- The process inverse to differentiation is called integration.

For example, $\frac{x^3}{3}, \frac{x^3}{2}+2, \frac{x^3}{2}-5$ are integrals of x^2.

Thus we can write

$\int f(x)\,dx = \phi(x) + c$

The symbol $\int f(x)\,dx$ is read as the indefinite integral of f(x)

with repect to x. $\int$ here is the integral sign.

f(x) is the integral sign

x is the variable of integration

dx is the differential of x or the element of integration

❑ STADARD FORMULAE

$(I)\ \frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = x^n, n \neq -1 \Rightarrow \int x^n\,dx = \frac{x^{n+1}}{n+1} + c$

$(ii)\ \frac{d}{dx}(e^x) = e^x \Rightarrow dx = e^x dx = e^x + c$

$(iii)\ \frac{d}{dx}\left(\frac{a^x}{\log_e a}\right) = a^x, a > 0, a \neq 1 \Rightarrow \int a^x dx = \frac{a^x}{\log_e a} + c$

$(iv)\ \frac{d}{dx}(\log_e x) = \frac{1}{x} \Rightarrow \int \frac{1}{x} dx = \log_e |x| + c$

$(v)\ \frac{d}{dx}(\log_a x) = \frac{1}{x}(\log_e) \Rightarrow \int \frac{1}{x}(\log_e e)\,dx = \log_a |x| + c$

$(vi)\ \frac{d}{dx}(\sin x) = \cos x \Rightarrow \int \cos x\,dx = \sin x + c$

$(vii)\ \frac{d}{dx}(\cos x) = -\sin x \Rightarrow \int \sin x\,dx = -\cos x + c$

$(viii)\ \frac{d}{dx}(\tan x) = \sec^c x \Rightarrow \int \sec^2 x\,dx = \tan x + c$

$(ix)\ \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x \Rightarrow \int \operatorname{cosec}^2 x\,dx = -\cot x + c$

$(x)\ \frac{d}{dx}(\sec x) = \sec x \tan x \Rightarrow \int \sec x \tan x\,dx = \sec x + c$

$(xi)\ \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x \cot x \Rightarrow \int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$

$(xii)\ \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \Rightarrow \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$

$(xiii)\ \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} dx = \cos^{-1} x + c$

$(xiv)\ \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \Rightarrow \int \frac{1}{1+x^2} dx = \tan^{-1} x + c$

$(xv)\ \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2} \Rightarrow \int -\frac{1}{1+x^2} dx = \cot^{-1} x + c$

$(xvi)\ \frac{d}{dx}(\sec^{2-1} x) = \frac{1}{x\sqrt{x^2-1}} \Rightarrow \int \frac{1}{x\sqrt{x^2-1}} dx = x + c$

$(xvii)\ \frac{d}{dx}(\operatorname{cosec}^{-1} x) = -\frac{1}{x\sqrt{x^2-1}} \Rightarrow \int -\frac{1}{x\sqrt{x^2-1}} dx = \operatorname{cosec}^{-1} x + c$

$$\int x^3 dx$$

solution (i)

Let $I = \int x^3 dx$

$$I = \frac{X^4}{4} + C$$

$$\int \frac{1+x^2}{x} dx$$

$$I = \int \frac{1+x^2}{x} dx$$

$$= \int \left(\frac{1}{x} + x\right) dx$$

$$= \int \frac{1}{x} dx + f\, x\, dx$$

$$= \log_e x + \frac{x^2}{2} + c$$

$$\int e^{x \log_e a} dx$$

solution

Let $I = \int e^{x \log_e a} dx$

$$= \int e^{\log_e a^x} dx = f\, a^x dx$$

$$= \frac{a^x}{\log_e a} + c$$

❑ METHOD OF SUBSITITUTION

- The method by which evaluating a integral by reducing it to standard form by substitution is called integration by substitution.

$$\int \frac{8x+13}{\sqrt{4x+7}}dx$$

solution (i)

$$I = \int \frac{8x+13}{\sqrt{4x+7}}dx$$

$$8x+13 = A(4x+7)+B$$

Comparing the coefficient of x and constant term, we get

A4=8⇒ A=2

and $7A+B\ =13 \Rightarrow B=-1$

$\therefore\ 8X+13=2(4X+7)-1$

$$\int (x^4+x^2+1)d$$

$$I=\int (x^4+x^2+1)d(x^2)$$

$$=\int (x^4+x^2+1)2x\,dx \quad \left[\because d(x^2)=2xdx\right]$$

$$=2\int (x^5+x^3+x)dx$$

$$=2\left[\frac{x^6}{6}+\frac{x^4}{4}+\frac{x^2}{2}\right]+c$$

$$=\frac{x^6}{3}+\frac{x^4}{2}+x^2+c$$

❑ METHOD OF FRACTION

- If the integrad is in the form of an algebraic fraction and the integral can not be evaluated by simple methods, we decompose the integrand into its partial fractions

$$\int \frac{3}{x^2+x} dx$$

partial fraction of $\frac{3}{x^2+x}$

$$\frac{3}{x^2+x} = \frac{3}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$

$3 = A(x+1)+Bx$

$3 = (A+B)x+A$

$A+B = 0$

$A = 3$

From equation i to ii we get $A = 3, B = -3$

$$\frac{3}{x^2+x} = \frac{3}{x} - \frac{3}{x+1}$$

$$\int \frac{3}{x^2+x} dx = 3\int \frac{dx}{x} - 3\int \frac{dx}{x+1}$$

$3\log(x) - 3\log(x+1) + c$

$3\log(x) - \log(x+1) + c$

$3\log\left(\frac{x}{x+1}\right) + c$

❑ INTEGRATION BY PARTS

- Integration by parts is a techniques that relates the integral of a product of two function. If f(x) and g(x) are two differentiable function of x. then

first function × integral of second function -

integral of [diffrential coefficient of the first × integral of second]

$I = \int x \sin 3x \, dx$

According to ILATE, algebric function taken as the first function

$$I = \int \underset{\substack{\uparrow \\ I}}{x} \ \underset{\substack{\uparrow \\ II}}{\sin 3x} \, dx$$

$$= x\int \sin 3x \, dx - \int\left\{\frac{d}{dx}x\int \sin 3x\right\}dx$$

$$= x\left(\frac{-\cos 3x}{3}\right) - \int \frac{\cos 3x}{3} dx$$

$$I = \frac{x \cos 3x}{3} + \frac{\sin 3x}{3.3} + c$$

$$= -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + c$$

$$= -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + c$$

$\int \sin(\log x)dx$

$I = \int \sin(\log x)dx$ (Put $\log x = t \Rightarrow x = e^t$)

$I = \int \sin t . e^t dt$

$$I = \frac{e^t}{1^2+1^2}(\sin t - \cos t) + c$$

$$I = \frac{e^t}{2}(\sin t - \cos t) + c$$

$$I = \frac{x}{2}[\sin(\log x) - \cos(\log x)] + c$$

$\int e^x\left(\log x + \frac{1}{x}\right)dx$

$$I = \int e^x\left(\log x + \frac{1}{x}\right)dx$$

$$I = \int \underset{\substack{\uparrow \\ II}}{e^x} \ \underset{\substack{\uparrow \\ I}}{\log x} \, dx + \int e^x \frac{1}{x} dx$$

$$I = \log x \, e^x - \int \frac{1}{x} e^x dx + \int e^x \frac{1}{x} dx$$

$$I = \log x \, e^x + c$$

❑ DEFINITE INTEGRALS

- If f(x) is a continuous function defined on an interval [a, b] and if (x) is the primitive of f(x). Then the definite integral of f(x) over [a, b] denoted by

$$\int_a^b f(x)dx$$

$$\int_a^b f(x)dx = [\phi(x)]_a^b = \phi(b) - \phi(a)$$

here a and b are called the lower limit and upper limit respectively.

$$\int_2^3 \frac{x}{x^2+1}dx$$

$$I = \int_2^3 \frac{x\,dx}{x^2+1}$$

$$I = \frac{1}{2}\int_2^3 \frac{2x\,dx}{x^2+1}$$

$$I = \frac{1}{2}[\log(x^2+1)]_2^3$$

$$I = \frac{1}{2}[\log(10) - \log(5)]$$

$$I = \frac{1}{2}\log\left(\frac{10}{5}\right) = \frac{1}{2}\log 2 \text{ (Ans)}$$

$$\int_0^{\pi/2} \cos^2 x\, dx$$

$$I = \frac{1}{2}\int_0^{\pi/2} \cos^2 x\, dx$$

$$I = \frac{1}{2}\int_0^{\pi/2} (1+\cos 2x)\, dx$$

$$I = \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2}$$

$$I = \frac{1}{2}\left[\left(\frac{\pi}{2} + \frac{\sin\pi}{2}\right) - \left(0 + \frac{\sin 0}{2}\right)\right] = \frac{\pi}{4}$$

❑ APPLICATION OF INTEGRATION

1. Average value of a function

$$f_{avg} = \frac{\int_a^b f(x)}{b-a} dx = \frac{1}{b-a}\int_a^b f(x)dx$$

2. Area under curve: Let y = f(x) be the curve area bounded by the curve f = f(x) between $x = x_1$ and $x = x_2$ is given by

$$A = \int_{x_1}^{x_2} y\, dx = \int_{x_1}^{x_2} f(x)\, dx$$