DETERMINANTS

Points to be covered in this topic

1. INTRODUCTION
2. TYPES OF DETERMINANTS
3. MINORS AND COFACTORS
4. PROPERTIES OF DETERMINANT
5. PRODUCTS OF DETERMINANT
6. APPLICATIONS OF DETERMINANT

❑ INTRODUCTION

- Every square matrix can be associated to an expression or a number which is known as its determinant.
- Let, A = $[a_{ij}]$ is a square matrix of order n, then determinant of A is denoted by det (A) or

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{31} & a_{32} & a_{3n} \end{pmatrix}$$

❑ TYPES OF MATRICES

❖ DETERMINANT OF A SQUARE MATRIX OF ORDER 2

- If A = $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ is a square matrix of order 2, then the determinant of A is denoted as

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11}a_{22} - a_{21}a_{12}$$

= Product of the diagonal elements minus the product of off diagonal elements

❖ DETERMINANT OF A SQUARE MATRIX OF ORDER 3

- The symbol $\Delta = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ is called a determinant of order 3 and its value is a number.

$$a_{11} = \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} - a_{11} = \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} + a_{13} = \begin{pmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$$

Example: Find the value the determinant

$$\Delta = \begin{pmatrix} 1 & 3 & -1 \\ -2 & 1 & 2 \\ 4 & 5 & 1 \end{pmatrix}$$

Expending Δ along the first row, we get

$$\Delta = 1\begin{pmatrix} 1 & 2 \\ 5 & 1 \end{pmatrix} - 3\begin{pmatrix} -2 & 2 \\ 4 & 1 \end{pmatrix} - 1\begin{pmatrix} -2 & 1 \\ 4 & 5 \end{pmatrix}$$

$$= 1(1\times1 - 5\times2) - 3(-2\times1 - 4\times2) - 1(-2\times5 - 4\times1)$$

$$= -9 + 30 + 14 = 35 \text{ (Ans)}$$

❑ MINOR AND COFACTORS

- Minors: If we leave the row and column passing through the elements a_{ij} than second order determinant thus obtained is called the minor of the element a_{ij} and is denoted by M_{ij}

The minor of element $a_{12} = \begin{pmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{pmatrix} = M_{12}$

The minor of element $a_{22} = \begin{pmatrix} a_{11} & a_{3} \\ a_{31} & a_{33} \end{pmatrix} = M_{22}$

We can write the expression in terms of minors as

$$\Delta = (-1)^{1+1} a_{11} M_{11} + (-1)^{1+2} a_{12} M_{12} + (-1)^{1+3} a_{13} M_{13} = a_{11} M_{11} - a_{12} M_{12} + a_{13} M_{13}$$

- Cofactors: when the minor M_{ij} is multiplied by $(-1)^{i+j}$ then this product is called the cofactors of a_{ij} and it is denoted by C_{ij}. For example

$$\text{Cofactor of } a_{ij} = C_{ij} = (-1)^{i+j} M_{ij}$$

Example

find the minors of the element of the determinant $\Delta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

M_{11} = Minor of the element a_{11} = 3

M_{12} = Minor of the element a_{12} = 1

M_{21} = Minor of the element a_{21} = 2

M_{22} = Minor of the element a_{22} = 1

❑ PROPERTIES OF DETERMINANTS

1. The value of a determinants does not change when rows and columns are interchanged.
2. If any two adjacent row or adjacent column of a determinant are interchanged the value of determinant is multiplied by -1
3. If any two rows or two column of a determinant are identical, the value of determinant is zero
4. If all element of one row or one column of a determinant are multiplied by some number k, the value of new determinant is k times of the value of the given determinant
5. If all element of a row or a column of a determinant are zero, the value of the determinant is zero
6. If in a determinant each element in any row or column consist of the sum of two terms then the determinant can be expressed as the sum of two determinants of the same order.
7. If two the element of a row or column of a determinant are added m times the corresponding elements of another row or coloumn the value of the determinant thus obtained is equal to the value of the original determinant.
8. If determinant becomes zero we put x = a in it, then (x-a) is factor of
9. In a determinant the sum of the product of the element of any row or any column with the cofactors of the elements of any other row or column is zero

Example

without expanding, show that $\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & b+c \end{vmatrix} = 0$

$$\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & b+c \end{vmatrix}$$

Applying $C_2 \rightarrow C_2 + C_3$, we get

$$\Delta = \begin{vmatrix} 1 & a+b+c & b+c \\ 1 & b+a+c & a+c \\ 1 & c+a+b & b+c \end{vmatrix}$$

Taking (a+b+c) as common from 2nd coloumn, we get

$$\Delta = (a+b+c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & a+c \\ 1 & 1 & b+c \end{vmatrix} = (a+b+c).0 = \text{(as } C_1 \text{ and } C_2 \text{ are identical)}$$

Example

without expanding, evaluating determinant $\begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$

$$\Delta = \begin{vmatrix} 41 & 1 & 5 \\ 79 & 7 & 9 \\ 29 & 5 & 3 \end{vmatrix}$$

Applying $C_1 \rightarrow C_1 + 8C_3$, we get

$$\Delta = \begin{vmatrix} 1 & 1 & 5 \\ 7 & 7 & 9 \\ 5 & 5 & 3 \end{vmatrix} = 0 \text{ (as } C_1 \text{ and } C_2 \text{ are identical)}$$

Example

$$show\ that\ \Delta = \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

$$\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$$

Taking a,b and c as common R_1, R_2, R_3 respectively, we get

$$\Delta = abc\begin{vmatrix} a & b & c \\ a & -b & c \\ a & b & -c \end{vmatrix}$$

Taking a,b and c as common from C1, C2, and C3 respectively, we get

$$\Delta = a^2b^2c^2\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Applying $C_2 \to C_2 + C_1$, $C_3 \to C_3 + C_1$

$$\Delta = a^2b^2c^2\begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix}$$

$$\Delta = a^2b^2c^2\left[-1\begin{vmatrix} 0 & 2 \\ 2 & 0 \end{vmatrix}\right] = 4a^2b^2c^2$$

❑ PRODUCT OF DETERMINANTS

- The determinants of same order can be multiplied by multiplication rule of matrices

$\Delta_1 = \begin{vmatrix} a & b & c \\ e & f & g \\ i & j & k \end{vmatrix}$ *and* $\Delta_2 = \begin{vmatrix} p & q & r \\ u & v & w \\ x & y & z \end{vmatrix}$ are two deteminant of same order then

$$\Delta_1\Delta_2 = \begin{pmatrix} a & b & c \\ e & f & g \\ i & j & k \end{pmatrix}\begin{pmatrix} p & q & r \\ u & v & w \\ x & y & z \end{pmatrix} = \begin{pmatrix} ap+bu+cx & ap+bv+cy & ar+bw+cz \\ ep+fu+gx & eq+fv+gy & er+fw+gz \\ ip+ju+kx & iq+jv+ky & ir+jw+kz \end{pmatrix}$$

❑ APPLICATION OF DETERMINANTS

1. Condition for collinearity of three point
2. Area of triangle
3. Equation of a line passing through two given points

MATRICES

Points to be covered in this topic

1. INTRODUCTION
2. TYPES OF MATRICES
3. OPERATIONS ON MATRICES
4. MATRIX MULTIPLICATION
5. TRANSPOSE OF A MATRIX
6. ADJOINT OF A MATRIX
7. INVERSE OF A MATRIX
8. SOLUTION OF SYSTEM OF LINER EQUATION
9. CAYLEY - HAMILTON THEOREM
10. APPLICATION OF MATRICES

❑ INTRODUCTION

- A set of mn numbers (real or complex), arranged in a rectangular
- formation (array or table) having m rows and n columns and enclosed by a square bracket [ ] is called m× n matrix (read “m by n matrix”) .

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{1n} \\ a_{21} & a_{22} & a_{2n} \\ a_{m1} & a_{m2} & a_{mn} \end{pmatrix}$$

❑ TYPES OF MATRICES

1. Row Matrix and Column Matrix: A matrix consisting of a single row is called a **row matrix or a row vector**, whereas a matrix having single column is called a **column matrix or a column vector.**

2. Null or Zero Matrix: A matrix in which **each element is "0"** is called a Null or Zero matrix. Zero matrices are generally denoted **by the symbol 0**. This **distinguishes zero matrix from the real number 0.**

3. Square matrix: A matrix A having same numbers of rows and columns is called a square matrix. A matrix A of order m x n can be written as $A_{m\times n}$. If m = n, then the matrix is said to be a square matrix. A square matrix of order n×n, is simply written as A_n.

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Particular cases of a square matrix

1. Diagonal matrix : A square matrix in which all elements are zero except those in the main or principal diagonal is called a diagonal matrix. Some elements of the principal diagonal may be zero but not all.

2. Scalar Matrix: A diagonal matrix in which all the diagonal elements are same, is called a scalar matrix

3. Identity Matrix or Unit matrix: A scalar matrix in which each diagonal element is 1(unity) is called a unit matrix. An identity matrix of order n is denoted by I_n.

4. Equal Matrices: Two matrices A and B are said to be equal if and only if they have the same order and each element of matrix A is equal to the corresponding element of matrix B

5. The Negative of a Matrix: The negative of the matrix $A_{m\times n}$, denoted by $-A_{m\times n}$, is the matrix formed by replacing each element in the matrix $A_{m\times n}$ with its additive inverse.

6. Transpose of a Matrix: A matrix which is formed by interchanging all the rows of a given into column and vice versa. The transpose of matrix A is written as A′ or A^T

❑ OPERATION ON MATRICES

1. Multiplication of a Matrix by a Scalar: If A is a matrix and k is a scalar (constant), then kA is a matrix whose elements are the elements of A , each multiplied by k

$$A = \begin{pmatrix} 4 & -3 \\ 8 & -2 \end{pmatrix} \text{then for a scaler K}$$

$$KA = \begin{pmatrix} 4K & -3K \\ 8K & -2K \end{pmatrix}$$

2. Addition and subtraction of Matrices: If A and B are two matrices of same order m×n then their sum A + B is defined as C, m×n matrix such that each element of C is the sum of the corresponding elements of A and B.

If $A = \begin{pmatrix} 2 & 3 \\ 5 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ Find A-B

Solution: here the matrice A and B are of same size so A - B possible

$$A - B = \begin{pmatrix} 2-1 & 3-2 \\ 5-3 & 1-1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}_{2\times 2}$$

size of (A-B) remains same as of A & B.

❑ MULTIPLICATION OF MATRICES

1. Multiplication of matrix by a scaler number: In order to multiply a matrix by a scaler number, multiply every entry by the given number.

Let $A = \begin{pmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{pmatrix}$ and $C = \begin{pmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{pmatrix}$ compute 2A-3B+4C

$$2A-3B+4C = 2\begin{pmatrix} -1 & 0 & 2 \\ 3 & 1 & 4 \end{pmatrix} - 3\begin{pmatrix} 0 & -2 & 5 \\ 1 & -3 & 1 \end{pmatrix} + 4\begin{pmatrix} 1 & -5 & 2 \\ 6 & 0 & -4 \end{pmatrix}$$

$$2A-3B+4C = \begin{pmatrix} -2 & 0 & 4 \\ 6 & 2 & 8 \end{pmatrix} - \begin{pmatrix} 0 & -6 & 15 \\ 3 & -9 & 3 \end{pmatrix} + \begin{pmatrix} 4 & -20 & 8 \\ 24 & 0 & -16 \end{pmatrix}$$

$$2A-3B+4C = \begin{pmatrix} 2 & -14 & 3 \\ 27 & 11 & -11 \end{pmatrix} \text{(Ans)}$$

2. Multiplication of two matrices: Two matrices A and B are said to be conformable for the product AB if the number of columns of A is equal to the number of rows of B.

Then the product matrix AB has the same number of rows as A and the same number of columns as B.

Thus the product of the matrices $A_{m\times p}$ and $B_{p\times n}$ is the matrix $(AB)_{m\times n}$.

EXAMPLE

if $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$

prove that $(A+B)^2 \neq A^2+2AB+B^2$

we have $A = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$

$A+B = \begin{pmatrix} 3 & 0 \\ 3 & 3 \end{pmatrix}$

and $(A+B)^2 = (A+B)(A+B) = \begin{pmatrix} 3 & 0 \\ 3 & 3 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} 9+0 & 0+0 \\ 9+9 & 0+9 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 18 & 9 \end{pmatrix}$

$A^2 = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1-2 & -1-3 \\ 9+9 & -2+9 \end{pmatrix} = \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix}$

$B^2 = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 4+1 & 2+0 \\ 2+0 & 1+0 \end{pmatrix} = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$

$AB = \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 2-1 & 1-0 \\ 4+3 & 2+0 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 7 & 2 \end{pmatrix}$

$2AB = 2\begin{pmatrix} 1 & 1 \\ 7 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 2 \\ 14 & 4 \end{pmatrix}$

$A^2+2AB+B^2 = \begin{pmatrix} -1 & -4 \\ 8 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 2 \\ 14 & 4 \end{pmatrix} + \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix}$

$= \begin{pmatrix} -1+5+2 & -4+2+2 \\ 8+14+2 & 7+4+1 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 24 & 12 \end{pmatrix}$

From equation 1 and equation 2 we can say that $(A+B)^2 \neq A^2+2AB+B^2$

if $\begin{bmatrix} 1 & 1 & x \end{bmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$, find x

we have $\begin{bmatrix} 1 & 1 & x \end{bmatrix}\begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$

$\begin{bmatrix} 1+0+2x & 0+2+x & 2+1+0 \end{bmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$

$\begin{bmatrix} 1+2x & 2+x & 3 \end{bmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 0$

$[(1+2x)\times 1+(2+x)\times 1+3\times 1] = 0$

$[6+3x] = 0$

$6+3x = 0$

$x = -2$

❑ TRANSPOSE OF MATRICES

- A matrix is formed by turning all the rows of given column and vice versa. The transpose of matrix A is written as A' and A^T
- **Properties of transpose matrix**

  I. $(A^T)^T = A$

  II. $(kA)^T = kA^T$

  III. $(A+B)^T = A^T + B^T$

  IV. $(AB)^T = B^TA^T$

If $A=\begin{pmatrix}1 & 3\\2 & 4\end{pmatrix}$ and $B=\begin{pmatrix}1 & 4\\2 & 5\end{pmatrix}$ verify that $(AB)^T = B^TA^T$

Here we have $A=\begin{pmatrix}1 & 3\\2 & 4\end{pmatrix}$ and $B=\begin{pmatrix}1 & 4\\2 & 5\end{pmatrix}$

$$AB=\begin{pmatrix}1 & 3\\2 & 4\end{pmatrix}\begin{pmatrix}1 & 4\\2 & 5\end{pmatrix}=\begin{pmatrix}7 & 19\\10 & 28\end{pmatrix}$$

$$(AB)^T=\begin{pmatrix}7 & 10\\19 & 28\end{pmatrix}$$

$$A^T=\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix} \text{ and } B^T=\begin{pmatrix}1 & 2\\4 & 5\end{pmatrix}$$

$$B^TA^T=\begin{pmatrix}1 & 2\\4 & 5\end{pmatrix}\begin{pmatrix}1 & 2\\3 & 4\end{pmatrix}=\begin{pmatrix}7 & 10\\19 & 28\end{pmatrix}$$

From that equation we can say that $(AB)^T = B^TA^T$

❑ ADJOINT OF MATRICES

- Let A is a square matrix, Adjoint of A is the transpose of the matrix of cofactors of the element of the matrix A. and is denoted by as Adj (A).

EXAMPLE

let $A=\begin{pmatrix}1 & 3\\2 & 5\end{pmatrix}$ Find adjoint of the matrix

Cofactor of 1 = 5

Cofactor of 3 = -2

Cofactor of 2 = -3

Cofactor of 5 = 1

Let B = Matrix of cofactors $=\begin{pmatrix}5 & -2\\-3 & 1\end{pmatrix}$

$$\text{Adjoint}(A) = B^T = \begin{pmatrix}5 & -2\\-3 & 1\end{pmatrix}^T = \begin{pmatrix}5 & -3\\-2 & 1\end{pmatrix}$$

❑ INVERSE OF MATRICES

- A square matrix A of order n is invertible, if they exists a square matrix B of the same order such that $AB = I_n = BA$
- **Properties of inverse**

1. A square matrix is invertible if it is non singular
2. If A is invertible $(A^{-1})^{-1} = A$
3. If A & B are invertible matrix of the same order then AB invertible and $(AB^{-1}) = B^{-1}A^{-1}$
4. The inverse of an invertible symmetric matrix is a symmetric matrix

EXAMPLE

Find adj (A) and A^{-1} for the matrix $A=\begin{pmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{pmatrix}$

we have $[A]=\begin{pmatrix}1 & 3 & 3\\ 1 & 4 & 3\\ 1 & 3 & 4\end{pmatrix}=1\neq 0$, so A is invertible

let c_{ij} is the cofactor of the element a_{ij} in A, then

$c_{11}=5,\ c_{12}=-1,\ c_{13}=-1,$

$c_{21}=-3, c_{22}=1,\ c_{23}=0,$

$c_{31}=-3, c_{32}=0,\ c_{33}=1$

$$\text{adj }(A)=\begin{pmatrix}7 & -1 & -1\\ -3 & 1 & 0\\ -3 & 0 & 1\end{pmatrix}=\begin{pmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{pmatrix}$$

$$A^{-1}=\frac{\text{Adj }(A)}{(A)}=\begin{pmatrix}7 & -3 & -3\\ -1 & 1 & 0\\ -1 & 0 & 1\end{pmatrix}$$

❑ SOLUTION OF SYSTEM OF LINER EQUATION

EXAMPLE

solve the system of homogenous liner equation

$2x+3y+4z = 0,\ x+y+z = 0,\ 2x\text{-}y+3 = 0$

Here we have

$$\Delta=\begin{pmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & -1 & 3 \end{pmatrix} = 2(3+1)-3(3-2)+4(-1-2)=8-3-12=-7\neq 0$$

$$\Delta_1=\begin{pmatrix} 0 & 3 & 4 \\ 0 & 1 & 1 \\ 0 & -1 & 3 \end{pmatrix}=0$$

$$\Delta_2=\begin{pmatrix} 2 & 0 & 4 \\ 1 & 0 & 1 \\ 2 & 0 & 3 \end{pmatrix}=0$$

$$\Delta_3=\begin{pmatrix} 2 & 3 & 0 \\ 1 & 1 & 0 \\ 2 & -1 & 0 \end{pmatrix}=0$$

$\Delta\neq 0,\ \Delta_1=0,\ \Delta_2=0$ and $\Delta_3=0$

There, the system of equations has trivial solution, i.e. $x = 0,\ y = 0,\ z = 0$.

❑ CAYLEY HAMILTON THEOREM

- Every square matrix satisfies its own characteristic equation

$\lambda^n + a_1\lambda^{n-1} + a_2\lambda^{n-2} + \dots\dots\dots\dots\dots\dots + a_n = 0$ is satisfied by A, therefore

$A^n + a_1A^{n-1} + a_2A^{n-2} + \dots\dots\dots\dots\dots\dots + a_nI = 0$

EXAMPLE

Verify cayley hamilton theorem for the matrix $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$

The characteristic equation of A is given by

$$\begin{pmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{pmatrix} = 0$$

$$-\lambda^3 + 6\lambda^2 - 9\lambda + 4 = 0$$

By cayley hamilton theorem we have

$$A^3 - 6A^2 + 9A\text{-}4I = 0$$

We have $A^2 = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}$

$$A^3 = \begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix}\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} = \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix}$$

Now $A^3 - 6A^2 + 9A\text{-}4I = \begin{pmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 21 & -21 & 22 \end{pmatrix} - 6\begin{pmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{pmatrix} + 9\begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix} - 4\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 22-36+18-4 & -21+30-0+0 & 21-30+9-0 \\ -21+30-9-0 & 22-36+18-4 & -21+30-9-0 \\ 21-30+9-0 & -21+30-9-0 & 22-36+18-4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Thus equation (1) is verified

❑ APPLICATION OF MATRICE IN SOLVING PHARMACOKINETIC EQUATION

1. Chemical reaction kinetics
2. Balancing of chemical equation
3. Chemical system